Mathematics : Arithmetic Progression

1) What is Arithmetic Progression?

Ans. : **Arithmetic Progression** is a progression in which the difference between any two consecutive terms is constant.

Examples 1) 1, 2, 3, 4, ...

2) 3, 7, 11, ...

3) a, a+d, a+2d .....[a+(n-1)d]

2) Write the general form of an AP

Ans. : a, (a+d), (a+2d), (a+3d) .....[a+(n-1)d]

3)Write the formula to find the Common Difference in AP

Ans. : $d = a_2 - a_1$ OR $d = a_3 - a_2$ OR $d = a_n - a_{(n-1)}$

4) What is a Finite Arithmetic Progression?

Ans. : An AP having finite number of terms is called finite Arithmetic Progression

Examples 1) 1, 2, 3, 4, ..., 10

2) 3, 7, 11, 15, 19

3) a, a+d, a+2d .....a+(n-1)d]

5) What is a Infinite Arithmetic Progression?

Ans. : An AP having infinite number of terms is called Infinite Arithmetic Progression

Examples 1) 2, 4, 6, 8, ...,

2) 5, 11, 17, 23, 29.....

3) a, a+d, a+2d .....

6) Write the n^{th} term of an AP

Ans. : $a_n = a + (n-1)d$

Where a = first term, d = Common difference, & n = number of terms

7)Last term of the AP is denoted by ..... or .....

Ans. : a_n OR L

8) If p and q are the p^{th} and q^{th} terms of an AP, Write the formula to find 'd'

Ans. : $d = \frac{a_p - a_q}{p - q}$

9) Write the formula to find the sum to n-terms of an AP?

Ans. : $S_n = \frac{n}{2}[2a + (n-1)d]$

10)Write the formula to find the sum to n-terms of an AP whose first term is 'a' & last term is "L"

Ans. : $Sn = \frac{n}{2}[a + L]$

11) Write the formula to find the sum of natural numbers(positive integers)?

Ans. : $S_n = \frac{n(n+1)}{2}$

Ex 1.1

2. Write the first four terms of the AP

(i)where a = 10 and d = 10

Ans. : First term a = 10

Second term $a_2 = a+d = 10 + 10 = 20$

Third term $a_3 = a_2+d = 20 + 10 = 30$

Forth term $a_4 = a_3+d = 30 + 10 = 40$

The first four terms of the AP are 10, 20, 30 and 40

(ii) where a = -2 and d = 0

Ans. : First term a = -2

Second term $a_2 = a+d = -2 + 0 = -2$

Third term $a_3 = a_2+d = -2 + 0 = -2$

Forth term $a_4 = a_3+d = -2 + 0 = -2$

The first four terms of the AP are -2, -2, -2 and -2

(iii) where a = 4 and d = -3

Ans. : First term a = 4

Second term $a_2 = a+d = 4 + (-3) = 4 - 3 = 1$

Third term $a_3 = a_2+d = 1 + (-3) = 1 - 3 = -2$

Forth term $a_4 = a_3+d = -2 + (-3) = -2 - 3 = -5$

The first four terms of the AP are 4, 1, -2 and -5

(iv) where $a = -1$ and $d = \frac{1}{2} = 0.5$

Ans. : First term $a = -1$

Second term $a_2 = a+d = -1 + 0.5 = -0.5$

Third term $a_3 = a_2+d = -0.50 + 0.5 = 0$

Forth term $a_4 = a_3+d = 0 + 0.5 = 0.5$

The first four terms of the AP are -1, -0.5, 0 and 0.5

(v) where $a = -1.25$ and $d = -0.25$

Ans. : First term $a = -1.25$

Second term $a_2 = a+d = -1.25 + (-0.25) = -1.25 - 0.25 = -1.5$

Third term $a_3 = a_2+d = -1.50 + (-0.25) = -1.50 - 0.25 = -1.75$

Forth term $a_4 = a_3+d = -1.725 + (-0.25) = -1.75 - 0.25 = -2.0$

The first four terms of the AP are -1.25, -1.5, -1.75 and 2.0

12) Write the next four terms of the following AP : 5, -1, 3. 7, ...

Ans. : $a = -5$

$d = a_2-a_1 = -1-(-5) = -1 + 5 = 4$

Fifth term $a_5 = a_4+d = 7 + 4 = 11$

Sixth term $a_6 = a_5+d = 11 + 4 = 15$

Seventh term $a_7 = a_6+d = 15 + 4 = 19$

Eighth term $a_8 = a_7+d = 19 + 4 = 23$

The Next four terms of the AP are 11, 15, 19 and 23

Ex 1.1

3. Find the first term and the Common Difference in the following AP's

(i) 3, 1, -1, -3 .....

Ans. : $a = 3$,

$d = a_2 - a_1 = 1 - 3 = -2$

(ii) -5, -1, 3, 7 .....

Ans. : $a = -5$

$d = a_2 - a_1$

$= -1 - (-5) = -1 + 5 = 4$

(iii) $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3}$.....

Ans. : $a = \frac{1}{3}$

$d = a_2 - a_1 = \frac{5}{3} - \frac{1}{3} = \frac{5-1}{3} = \frac{4}{3}$

(iv) 0.6, 1.7, 2.8, 3.9.....

Ans. : $a = 0.6$

$d = a_2 - a_1 = 1.7 - 0.6 = 1.1$

Ex 1.1

4. Which of the following are APs?

(i)2, 4, 8, 16 .....

Ans. : $a_2 - a_1 = 4 - 2 = 2$

$a_3 - a_2 = 8 - 4 = 4$

$a_4 - a_3 = 16 - 8 = 8$

Since the difference between any two consecutive terms are not constant,

Therefore it is not in AP

(v) $3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2}$.....

Ans. : $a_2 - a_1 = 3+\sqrt{2} - 3 = \sqrt{2}$

$a_3 - a_2 = 3+2\sqrt{2} - (3+\sqrt{2}) = 3+2\sqrt{2} - 3 - \sqrt{2} = \sqrt{2}$

$a_4 - a_3 = 3+3\sqrt{2} - (3+3\sqrt{2}) = 3+3\sqrt{2} - 3 - 2\sqrt{2} = \sqrt{2}$

Since the difference between any two consecutive terms are constant

Therefore it is not in AP

(vii) 0, -4, -8, -12 .....

Ans. : $a_2 - a_1 = -4 - 0 = -4$

$a_3 - a_2 = -8 - (-4) = -8 + 4 = -4$

$a_4 - a_3 = -12 - (-8) = -12 + 8 = -4$

Since the difference between any two consecutive terms are constant

Therefore it is in AP

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13) Find the 15^{th} term of the AP

Ans. : $a_{15} = a + 14\,d$

14) Find the 10^{th} term of an AP 2, 7, 12, ...

Ans. : $a = 2$ $\qquad a_n = a + (n - 1)\,d$

$d = a_2 - a_1$ $\qquad a_{10} = a + 9d$

$= 7 - 2$ $\qquad a_{10} = 2 + 9(5)$

$d = 5$ $\qquad a_{10} = 2 + 45 = 47$

$n = 10$ $\qquad$ The 10^{th} term of an AP is 47

$a_{10} = ?$

15) Find the 30^{th} term of the AP 10, 7, 4, ...

Ans. : $a = 10$ $\qquad a_n = a + (n - 1)\,d$

$d = a_2 - a_1$ $\qquad a_{30} = a + 29d$

$d = 7 - 10 = -3$ $\qquad a_{30} = 10 + 29(-3)$

$n = 30$ $\qquad a_{30} = 10 - 87 = -77$

$a_{30} = ?$ $\qquad$ The 30^{th} term of an AP is -77

16) Find the 11^{th} term of the AP $-3, -\frac{1}{2}, 2$.....

Ans. : -3, -0.5, 2 ..... $\qquad a_n = a + (n - 1)\,d$

$a = -3$ $\qquad a_{11} = a + 10d$

$d = a_2 - a_1 = -0.5 - (-3) = -0.5 + 3$ $\qquad a_{11} = -3 + 10(2.5)$

$d = 2.5$ $\qquad a_{11} = -3 + 25.0$

$n = 11$ $\qquad a_{11} = 22$

$a_{11} = ?$ The 11^{th} term of an AP is 22

17) Find the n^{th} term of AP : 3, 7, 11, 15, ...

Ans. : $a = 3$ $a_n = a + (n - 1)\,d$

$d = a_2 - a_1$ $a_n = 3 + (n - 1)(4)$

$d = 7 - 3 = 4$ $a_n = 3 + 4n - 4$

$n = n$ $a_n = 4n - 1$

$a_n = ?$

18) Which term of the AP : 21, 18, 15, ...is -81?

Ans. : $a = 21$ $a_n = a + (n - 1)\,d$

$d = a_2 - a_1 = 18 - 21 = -3$ $-81 = 21 + (n - 1)(-3)$

$n = ?$ $-81 - 21 = (n - 1)(-3)$

$a_n = -81$ $-102 = (n - 1)(-3)$

$$(n-1) = \frac{-102}{-3}$$

$n - 1 = 34$

$n = 34 + 1$

$n = 35$

The 35th term is -81

19) Check whether 301 is a term of the list of numbers 5, 11, 17, 23.....

Ans. : Let $a = 5$ $a_n = a + (n - 1)\,d$

$d = 11 - 5 = 6$ $301 = 5 + (n - 1)(6)$

$a_n = 301$ $301 = 5 + 6n - 6$

$301 = 6n - 1$

$301 + 1 = 6n$

$$n = \frac{302}{6}$$

$n = 50.33$

Since the value of n is not a natural number

$\therefore$ 301 is not a term of the given list of terms

20) Determine the AP whose 3rd term is 5 and the 7th term is 9.

Ans. : $a_3 = a + 2d$ $\quad a_7 = a + 6d$

$5 = a + 2d$ .....(1) $\quad 9 = a + 6d$ .....(2)

Subtract Eqn.(1) from Eqn.(2)

$a + 6d = 9$ $\quad a + 2d = 5$

$a + 2d = 5$ $\quad a + 2(1) = 5$

(-) (-) (-) $\quad a + 2 = 5$

$4d = 4$ $\quad a = 3$

$d = 1$ $\quad$ The required AP is 3, 4, 5, 6, ...

Ex 1.2

1. (i) In an AP, If $a = 7, d = 3, n = 8$, find a_n

Ans. : $a_n = a + (n - 1) d$

$a_8 = 7 + (8 - 1) 3$

$a_8 = 7 + 7(3)$

$a_8 = 7 + 21 = 28$

The 8th term of an AP is 28

(ii) In an AP, If $a = -18, n = 10, a_n = 0$, find d

Ans. : $a = -18$ $\quad a_n = a + (n - 1) d$

$n = 10$ $\quad 0 = -18 + (10 - 1) d$

$a_n = 0$ $\quad 18 = 9d$

$d = ?$ $\quad d = 2$

The Cd is 2

(iii) In an AP, If $d = -3, n = 18, a_n = -5$, find a

Ans. : $d = -3$ $\quad a_n = a + (n - 1) d$

$n = 18$ $\quad -5 = a + (18 - 1) (-3)$ -

$a_n = -5$ $\quad -5 = a - 17(3)$

$a = ?$ $\quad -5 = a - 51$

$-5 + 51 = a$

$a = 46$

The first term of the AP is 46

(iv) In an AP, If $a = -18.9$, $d = 2.5$, $a_n = 3.6$, find n

Ans. : $a = -18.9$ $\quad a_n = a + (n - 1)\, d$

$d = 2.5$ $\quad 3.6 = -18.9 + (n - 1)\,(2.5)$

$a_n = 3.6$ $\quad 3.6 + 18.9 = (n - 1)\,(2.5)$

$n = ?$ $\quad 22.5 = (n - 1)\,(2.5)$

$$(n-1) = \frac{22.5}{2.5}$$

$n - 1 = 9$

$n = 9 + 1$

$n = 10$

There are 10 terms in the AP

(v) In an AP, If $a = 3.5$, $d = 0$, $n = 105$ find a_n

Ans. : $a = 3.5$ $\quad a_n = a + (n - 1)\, d$

$d = 0$ $\quad a_n = 3.5 + (105 - 1)\, 0$

$n = 105$ $\quad a_n = 3.5 + 0$

$a_n = ?$ $\quad a_n = 3.5$

The 105^{th} term of an the AP is 3.5

Ex 1.2

4. Which term of the AP 3, 8, 13, 18, ...is 78?

Ans. : $a = 3$ $\quad a_n = a + (n - 1)\, d$

$d = a_2 - a_1$ $\quad 78 = 3 + (n - 1)(5)$

$= 8 - 3$ $\quad 78 - 3 = (n - 1)(5)$

$d = 5$ $\quad 75 = (n - 1)(5)$

$n = ?$ $\quad n - 1 = \frac{75}{5}$

$a_n = 78$ $\quad$ $n - 1 = 15$

$n = 15 + 1$

$n = 16$

The 16^{th} term is 78

5. (i) Find the number of terms of the AP : 7, 13, 19.....205?

Ans. : $a = 7$ $\quad$ $a_n = a + (n - 1)\,d$

$d = a_2 - a_1 = 13 - 7 = 6$ $\quad$ $205 = 7 + (n - 1)(6)$

$n = ?$ $\quad$ $205-7 = (n - 1)(6)$

$a_n = 205$ $\quad$ $198 = (n - 1)(6)$

$n - 1 = \frac{198}{6}$

$n - 1 = 33$

$n = 33 + 1 = 34$

The 34th term is 205

5. (ii) Find the number of term of the AP $18, 15\frac{1}{2}, 13, \ldots\ldots\ldots\ldots -47$?

Ans. : $a = 18$ $\quad$ $a_n = a + (n - 1)\,d$

$d = a_2 - a_1 = 15.5 - 18 = -2.5$ $\quad$ $-47 = 18 + (n - 1)(-2.5)$

$n = ?$ $\quad$ $-47-18 = (n - 1)(-2.5)$

$a_n = -47$ $\quad$ $-65 = (n - 1)(-2.5)$

$n - 1 = \frac{-65}{-2.5}$

$n - 1 = 26$

$n = 26 + 1$

$n = 27$

The 27th term is -47

6. Check whether -150 is a term of the AP : 11, 8, 5, 2.....

Ans. : Let $a = 11$ $\quad$ $a_n = a + (n - 1)\,d$

$d = 8 - 11 = -3$ $\quad$ $-150 = 11 + (n - 1)(-3)$

$a_n = -150$ $\quad$ $-150 = 11-3n+ 3$

$-150-14 = 3n$

$$n = \frac{-164}{3}$$

$n = 54.66$

Since the value of n is not a natural number

therefore -150 is not a term of the given list of terms

7. Find the 31st term of the AP whose 11th term is 38 and the 16th term is 73.

Ans. : $a_{11} = a + 10d$ $\qquad$ $a_{16} = a + 15d$

$38 = a + 10d$ .....(1) $\qquad$ $73 = a + 15d$ .....(2)

Subtract Eqn.(1) from Eqn.(2)

$a + 15d = 73$

$-a + 10d = 38$

(-) (-) (-)

$5d = 35$

$d = 7$

$a + 10d = 38$

$a + 10(7) = 38$

$a = 38 - 70$

$a = -32$

$a_{31} = a + 30d$

$a_{31} = -32 + 30(7)$

$a_{31} = -32 + 210$

$a_{31} = 178$

$\therefore$ The 31st term is 178

8. An AP consists of 50 terms of which 3rd term is 12 and the last term is 106. Find the 29th term?

Ans. : $a_3 = a + 2d$ $\qquad$ $a_{50} = a + 49d$

$12 = a + 2d$ .....(1) $\qquad$ $106 = a + 49d$ .....(2)

Subtract Eqn.(1) from Eqn.(2)

$a + 49d = 106$

$a + 2d = 12$

(-) (-) (-)

$47d = 94$

$d = 2$

$a + 2d = 12$

$a + 2(2) = 12$

$a + 4 = 12$

$a = 12 - 4$

$a = 8$

$a_{29} = a + 28d$

$a_{29} = 8 + 56$

$a_{29} = 8 + 28(2)$

$a_{29} = 64$

$\therefore$ The 29th term is 64

9. If the 3rd and 9th terms of an AP are 4 and -8 respectively, which term of this AP is Zero?

Ans. : $a_3 = a + 2d$

$4 = a + 2d$ .....(1)

$a_9 = a + 8d$

$-8 = a + 8d$ .....(2)

Subtract Eqn.(1) from Eqn.(2)

$a + 8d = -8$

$a + 2d = 4$

(-) (-) (-)

$6d = -12$

$d = -2$

$a + 2d = 4$

$a + 2(-2) = 4$

$a - 4 = 4$

$a = 4 + 4$

$a = 8$

$a_n = a + (n - 1)d$

$0 = 8 + (n - 1)(-2)$

$-8 = (n - 1)(-2)$

$n - 1 = \frac{-8}{-2}$

$n - 1 = 4$

$n = 4 + 1$

$n = 5$

$\therefore$ The 5th term is zero

10. The 17^{th} term of an AP exceeds its 10^{th} term by 7. Find common difference

Ans. : $a_{17} = a_{10} + 7$

$a + 16d = a + 9d + 7$

$16d - 9d = 7$

$7d = 7$

$d = 1$ $\therefore$ The common difference is 1

11. Which term of the AP: 3, 15, 27, 39..... will be 132 more than its 54th term?

Ans. : 3, 15, 27, 39.....

$a = 3,$

$d = 15 - 3 = 12$

$a_n = a_{54} + 132$

$\cancel{a} + (n - 1)\, d = \cancel{a} + 53d + 132$

$(n - 1)\, d = 53d + 132$

$(n - 1)\,(12) = 53(12) + 132$

$12n - 12 = 636 + 132$

$12n - 12 = 768$

$12n = 768 + 12$

$12n = 780$

$n = \frac{780}{12} = 65$

65^{th} term will be 132 more than its 54^{th} term

12. Two APs have the same common difference. The difference between their 100^{th} terms is 100. What is the difference between their 1000 terms?

Ans. : $[a_1]_{100} - [a_2]_{100} = 100$

$a_1 + 99d - [a_2 + 99d] = 100$

$a_1 + 99d - a_2 - 99d = 100$

$a_1 - a_2 = 100$ .....(1)

$[a_1]_{1000} - [a_2]_{1000}$

$= a_1 + 999d - [a_2 + 999d]$

$= a_1 + 999d - a_2 - 999d$

$= a_1 - a_2 = 100$

13. How many three-digit numbers are divisible by 7?

Ans. : The three digit numbers which are divisible by 7 are 105 , 112, 119, ...994

$a = 105$ $\quad a_n = a + (n - 1)\, d$

$d = a_2 - a_1$ $\quad 994 = 105 + (n - 1)(7)$

$= 112 - 105$ $\quad 994 = 105 + 7n - 7$

$d = 7$ $\quad 994 = 7n + 98$

$n = ?$ $\quad 7n = 994 - 98$

$a_n = 994$ $\quad 7n = 896$

$$n = \frac{896}{7} = 128$$

There are 128 three digit numbers

14. How many multiples of 4 lie between 10 & 250?

Ans. : 12 , 16, 20, ...248 $\quad a_n = a + (n - 1)\, d$

$a = 12$ $\quad 248 = 12 + (n - 1)(4)$

$d = a_2 - a_1 = 16 - 12 = 4$ $\quad 248 = 12 + 4n - 4$

$n = ?$ $\quad 248 = 4n + 8$

$a_n = 248$ $\quad 4n = 248 - 8$

$4n = 240$

$$n = \frac{240}{4}$$

$n = 60$

There are 60 numbers

15. For what value of 'n' are the n^{th} terms of two APs 63, 65, 67 .....& 3,10,17.....equal?

Ans.: Let $a_1 = 63$ $\quad a_2 = 3$ $\quad [a_n]_1 = [a_n]_2$

$d_1 = 65 - 63$ $\quad d_2 = 10 - 3$ $\quad a_1 + (n-1)d_1 = a_2 + (n-1)d_2$

$d_1 = 2$ $\quad d_2 = = 7$ $\quad 63 + (n-1)2 = 3 + (n-1)7$

$63 + 2n - 2 = 3 + 7n - 7$

$61 + 2n = 7n - 4$

$61 + 4 = 7n - 2n$

$5n = 65$

$n = 13$

16. Determine the AP whose 3rd term is 16 & 7th term exceeds the 5th term by 12?

Ans. : $a_7 = a_5 + 12$

$a+6d = a+4d + 12$

$6d - 4d = 12$

$2d = 12$

$d = 6$

$a_3 = a + 2d$

$16 = a + 2d$

$16 = a + 2(6)$

$16 - 12 = a$

$a = 4$

The required AP is 4, 10, 16.....

17. Find the 20th term from the last term of the AP 3, 8, 13, ...253

Ans. : Let $a = 253$

$d = a_1 - a_2$

$d = 3 - 8 = -5$

$n = 20$

$a_n = ?$

$a_n = a + (n - 1)\, d$

$a_{20} = a + 19d$

$a_{20} = 253 + 19(-5)$

$a_{20} = 253 - 95 = 158$

the 20th term from the last term of the AP is 158

18. The sum of the 4th and 8th terms of an AP is 24 and the sum of 6th and 10th terms is 44. find the first three terms of the AP?

Ans. : $a_4 + a_8 = 24$

$a + 3d + a + 7d = 24$

$2a + 10d = 24$ .....÷2

$a + 5d = 12$ .....(1)

$a_6 + a_{10} = 44$

$a + 5d + a + 9d = 44$

$2a + 14d = 44$ .....÷2

$a + 7d = 22$ .....(2)

Subtract Eqn.(1) from Eqn.(2)

$a + 7d = 22$

$a + 5d = 12$

(-) (-) (-)

$2d = 10$

$d = 5$

$a + 7d = 22$

$a + 7(5) = 22$

$a + 35 = 22$

$a = 22-35$

$a = -13$

The first three terms are -13, -8 & -3

19. Subba Rao started work in 1995 at an annual salary of Rs.5000 and received an increment of Rs. 200 each year. In which year did his income reach Rs. 7000?

Ans. : $a = 5000$ $\quad a_n = a + (n - 1)d$

$d = 200$ $\quad 7000 = 5000 + (n - 1)200$

$a_n = 7000$ $\quad 7000 = 5000 + 200n - 200$

$n = ?$ $\quad 7000 = 4800 + 200n$

$7000 - 4800 = 200\ n$

$200n = 2200$

$n = 11$

In 2006 he will receive 7000 salary

20. Ramkali saved Rs.5 in the first week of a year and then increased her weekly saving by Rs.1.75. If in the nth week. her weekly saving become Rs. 20.75. Find n

Ans. : $a = 5$ $\quad a_n = a + (n - 1)d$

$d = 1.75$ $\quad 20.75 = 5 + (n - 1)(1.75)$

$n = ?$ $\quad 20.75 - 5 = (n - 1)(1.75)$

$a_n = 20.75$ $\quad 15.75 = (n - 1)(1.75)$

$$n - 1 = \frac{15.75}{1.75}$$

$n - 1 = 9$

$n = 9 + 1$

$n = 10$

Ex 1.3

I. Find the sum of the following APs

(i) 2, 7, 12.....to 10 terms

Ans. : $a = 2$

$d = a_2 - a_1 = 7 - 2 = 5$

$n = 10$

$S_{10} = ?$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{10} = \frac{10}{2}[2(2) + (10-1)5]$$

$S_{10} = 5\ [\ 4 + 9(5)]$

$S_{10} = 5\ (4 + 45)$

$S_{10} = 5\ (49) = 245$

The sum to 10 terms is 245

(ii) -37, -33, -29.....to 12 terms

Ans. : a =-37

d = a_2 - a_1 = -33-(-37)

d = -33+37 = 4

n = 12

S_{12} = ?

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{12} = \frac{12}{2}[2(-37) + (12-1)(4)]$$

$S_{12} = 6\ [-74 + 11(4)]$

$S_{10} = 6\ (-74 + 44)$

$S_{10} = 6\ (-30) = -180$

The sum to 12 terms is -180

(iii) 0.6, 1.7, 2.8,.....100 terms

Ans. : a =0.6

d = a_2 - a_1

= 1.7-0.6

d = 1.1

n = 100

S_{100} = ?

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{100} = \frac{100}{2}[2(0.6) + (100-1)(1.1)]$$

$S_{100} = 50\ [1.2 + 99(1.1)]$

$S_{100} = 50\ (1.2 + 108.9)$

$S_{100} = 50\ (110.1) = 5505$

The sum to 100 terms is 5505

(iv) Find the sum of : $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}$,.....to 11 terms

Ans. : a = $\frac{1}{15}$

$$d = \frac{1}{12} - \frac{1}{15} = \frac{5-4}{60} = \frac{1}{60}$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_n = \frac{11}{2}\left[2\left(\frac{1}{15}\right) + (11-1)\left(\frac{1}{60}\right)\right]$$

n = 11

S_{11} = ?

$$S_n = \frac{11}{2}[\frac{2}{15} + (10)\left(\frac{1}{60}\right)]$$

$$S_n = \frac{11}{2}[\frac{2}{15} + \frac{1}{6}]$$

$$S_{11} = \frac{11}{2}[\frac{2(2)+5}{30}]$$

$$S_{11} = \frac{11}{2} x \frac{9}{30}$$

$$S_{11} = \frac{11}{2} x \frac{3}{10} = \frac{33}{20}$$

The sum to 11 terms is $\frac{33}{20}$

2. Find the sum of :

(i) $7 + 10\frac{1}{2} + 14 ..... + 84$

Ans. : a = 7

d = $a_2 - a_1$ = 10.5 - 7 = 3.5

n = ?

a_n = L= 84

S_n = ?

$a_n = a + (n - 1) d$

84 = 7 + (n - 1) (3.5)

84 -7 = (n - 1) (3.5)

77 = (n - 1) (3.5)

$$n - 1 = \frac{77}{3.5}$$

n-1 = 22

n = 22+1

n = 23

$$Sn = \frac{n}{2}[a + L]$$

$$S_{23} = \frac{23}{2}[7 + 84]$$

$$S_{23} = \frac{23}{2}[91]$$

S_{23}= 1046.5

The sum to 23 terms is 1046.5

(ii) 34+32+30+.....+10

Ans. : $a = 0.6$

$d = a_2 - a_1 = 32 - 34 = -2$

$n = ?$

$a_n = 1$

$S_n = ?$

$a_n = a + (n - 1)\, d$

$10 = 34 + (n - 1)(-2)$

$10 = 34 - 2n + 2$

$10 = 36 - 2n$

$10 - 36 = -2n$

$-2n = -26$

$n = 13$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{13} = \frac{13}{2}[2(34) + (13-1)(-2)$$

$$S_{13} = \frac{13}{2}[68 + (12)(-2)]$$

$$S_{13} = \frac{13}{2}[68 - 24]$$

$$S_{13} = \frac{13}{2}[44]$$

$S_{13} = 13 \times 22$

$S_{13} = 286$

The sum to 13 terms is 286

(iii) $-5 + (-8) + (-11) + \ldots\ldots\ldots\ldots + (-230)$

Ans. : $a = -5$

$d = a_2 - a_1 = -8 - (-5)$

$= -8 + 5$

$d = -3$

$n = ?$

$a_n = L = -230$

$S_n = ?$

$a_n = a + (n - 1)\, d$

$-230 = -5 + (n - 1)(-3)$

$-230 = -5 - 3n + 3$

$-230 = -2 - 3n$

$-230 + 2 = -3n$

$-228 = -3n$

$n = 76$

$$Sn = \frac{n}{2}[a + L]$$

$$S_{76} = \frac{76}{2}[-5 + (-230)]$$

S_{76} = 38 (-5-230)

S_{76} = 38 (-235)

S_{76} = -8930

The sum to 76 terms is -8930

3. (i) In an AP given a = 5, d = 3, a_n = 50, find n and S_n

Ans. : a_n = a + (n-1)d

50 = 5 + (n-1)3

50 = 5 + 3n-3

50 = 2 + 3n

50 - 2 = 3n

3n = 48

n = 16

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{16} = \frac{16}{2}[2(5) + (16-1)(3)]$$

S_{16} = 8 [10 +45]

S_{16} = 8 [55]

S_{16} = 440

3. (ii) In an AP given a = 7, a_{13} = 35, find d and S_{13}

Ans. : a_n = a + (n-1)d

35 = 7 + (13-1)d

35 -7= 12 d

28 = 12d

$d = \frac{28}{12} = \frac{7}{3}$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{13} = \frac{13}{2}[2(7) + (13-1)(\frac{7}{3})]$$

$$S_{13} = \frac{13}{2}[14 + 28]$$

$$S_{13} = \frac{13}{2}[42]$$

S_{13} = 13 x 21

S_{13} = 273

3. (iii) In an AP given, a_{12} = 37, d = 3 find a and S_{12}

n = 12, d = 3, a_{12} = 37

$a_{12} = 37$

$a + 11d = 37$

$a + 11(3) = 37$

$a + 33 = 37$

$a = 37 - 33$

$a = 4$

$S_n = \frac{n}{2}[a + L]$

$S_{12} = \frac{12}{2}[4 + 37]$

$S_{12} = 6\ [41]$

$S_{12} = 246$

3. (iv) In an AP given, $a_3 = 15$, $S_{10} = 125$ find d and a_{10}

Ans. : $a_3 = 15$

$a + 2d = 15$ .....x 2

$2a + 4d = 30$ .....(1)

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{10} = \frac{10}{2}[2a + (10-1)d]$

$125 = 5[\ 2a + 9d]$

$\frac{125}{5} = [\ 2a + 9d]$

$25 = [\ 2a + 9d]$

Sub (2) from (1)

$2a + 4d = 30$

$2a + 9d = 25$

(-) (-) (-)

$5d = -5$

$d = -1$

$2a + 9d = 25$.....(2)

$a + 2d = 15$

$a + 2(-1) = 15$

$a - 2 = 15$

$a = 17$

$a_{10} = a + 9d$

$a_{10} = 17 + 9(-1)$

$a_{10} = 17 - 9$

$a_{10} = 8$

Therefore the common difference is -1 and the 10th term is 8

3.(v) In an AP given, $d = 5$, $S_9 = 75$ find a and a_9

Ans. : $S_n = \frac{n}{2}[2a + (n-1)d]$

$75 = \frac{9}{2}[2a + (9-1)(5)]$

75 x 2= 9 [2a + 40]

150 = 18a + 360

150 - 360 = 18a

18 a = -210

$a = \frac{-210}{18}$

$a = \frac{-35}{3}$

$a_9 = a + 8d$

$a_9 = \frac{-35}{3} + 8(5)$

$a_9 = \frac{-35}{3} + 40$

$a_9 = \frac{-35+120}{3}$

$a_9 = \frac{85}{3}$

3. (vi) In an AP given, a = 2, d = 8 and $S_n = 90$ find n and a_n

Ans : $S_n = \frac{n}{2}[2a + (n-1)d]$

$90 = \frac{n}{2}[2(2) + (n-1)(8)]$

$90 = \frac{n}{2}x2[2 + (n-1)(4)]$

90 = n[2 + 4n - 4]

90 = n[4n - 2]

$90 = 4n^2 - 2n$ .....(÷2)

$45 = 2n^2 - n$

$2n^2 - n - 45 = 0$

$2n^2 - 10n + 9n - 45 = 0$

2n(n - 5) + 9(n - 5) = 0

(n - 5) (2n + 9) = 0

(n - 5) = 0 or (2n + 9) = 0

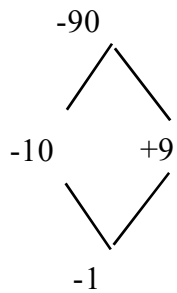
$n = 5$ or $n = \frac{-9}{2}$

$a_5 = a + 4d$

$a_5 = 2 + 4(8)$

$a_5 = 2 + 32$

$a_5 = 34$



3. (vii) In an AP given,a=8, a_n = 62 & S_n=210 , find n and d

Ans. : $a_n = 62$

$a + (n-1)d = 62$

$8 + (n-1)d = 62$

$(n-1)d = 54$....(1)

$S_n = \frac{n}{2}[2a + (n-1)d]$

$210 = \frac{n}{2}[2(8) + (n-1)d]$

$420 = n\ [\ 16 + 54]$

$420 = n\ [70]$

$n = 6$

But

$(n-1)d = 54$

$(6-1)d = 54$

$d = \frac{54}{5}$

3.(viii) In an AP given, a_n = 4 , d=2, & S_n = -14 find n and a

Ans. : $a_n = 4$

$a + (n-1)d = 4$

$a + (n - 1)\ 2 = 4$

$a + 2n - 2 = 4$

$a + 2n = 4 + 2$

$a = 6 - 2n$

$a + 2n = 6$ ....(1)

$S_n = \frac{n}{2}[2a + (n-1)d]$

$-14 = \frac{n}{2}[2a + (n-1)(2)]$

$-28 = n\ [\ 2(6-2n) + (n-1)2]$

$-28 = n[\ 12-4n\ + 2n - 2]$

$-28 = n\ [-2n+10]$

$-28 = -2n^2 + 10n$

$2n^2 - 10n - 28 = 0$ .....(÷2)

$n^2 - 5n - 14 = 0$

$n^2 - 7n + 2n - 14 = 0$

$n(n-7)+2(n-7) = 0$

$(n-7) = 0$ or $n+2 = 0$

$n = 7$ or $n = -2$

But $a = 6 - 2n$

$a = 6 - 2(6)$

$a = 6 - 12$

$a = -8$

3. (ix) In an AP given, a = 3, n = 8 and S_n = 192 find d

Ans. : $S_n = \frac{n}{2}[2a + (n-1)d]$

$192 = \frac{8}{2}[2(3) + (8-1)d]$

$192 = 4\ [\ 6 + 7d]$

$\frac{192}{4} = [\ 6 + 7d]$

$48 = 6 + 7d$

$48 - 6 = 7d$

$7d = 42$

$d = 6$

3. (x) In an AP given, L= 28, and S_n = 144 and there are total 9 terms. Find a

Ans. : $S_n = \frac{n}{2}[a + L]$

$144 = \frac{9}{2}[a + 28]$

$288 = 9\ [\ a + 28]$

$\frac{288}{9} = [\ a + 28]$

$a + 28 = 32$

$a = 32 - 28$

$a = 4$

4. How many term of the AP : 9, 17, 25..... must be taken to give a sum of 636?

Ans.: a = 9

$S_n = \frac{n}{2}[2a + (n-1)d]$

$d = a_2 - a_1$

$636 = \frac{n}{2}[2(9) + (n-1)(8)]$

d= 17 - 9 = 8

$636 = \frac{n}{2} x 2[(9) + (n-1)(4)]$

n = ?

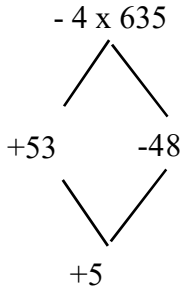
$636 = n[9 + 4n - 4]$

S_n = 636

$636 = n[\ 4n + 5]$

$636 = 4n^2 + 5n$

$4n^2 + 5n - 636 = 0$



$4n^2 + 53n - 48n - 636 = 0$

$n[4n + 53] -12[4n + 53] = 0$

$[4n + 53] [n -12] = 0$

$4n + 53 = 0$ & $n -12 = 0$

$4n = -53$ & $n = 12$

$n = -\frac{53}{4}$ & $n = 12$

There are 12 terms in the series

5. The first term of an AP is 5, the last term is 45 and the sum is 400. Find the number of terms and common difference?

Ans.: $a = 5$ $\quad a_n = a + (n - 1)d$

$d = ?$ $\quad 45 = 5 + (n - 1)d$

$n = ?$ $\quad 45 - 5 = (n - 1)d$

$a_n = L = 45$ $\quad (n - 1)d = 40$ .....(1)

$s_n = 400$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$400 = \frac{n}{2}[2(5) + (n-1)d]$$

$800 = n [10 +40]$

$800 = n [50]$

$n = 16$

But $\quad a_n = a + (n - 1)d$

$a_{16} = a + 15d$

$45 = 5 + 15d$

$15 d = 40$

$45 - 5 = 15d$

$$d = \frac{40}{15} = \frac{8}{3}$$

6. The first and the last term of an AP are 17 and 350 respectively. If common difference is 9. How many terms are there and what is their sum?

Ans.: $a = 17$

$d = 9$

$n = ?$

$S_n = ?$

a_n (L)$= 350$

$a_n = a + (n-1)d$

$350 = 17 + (n-1)9$

$350 = 17 + 9n-9$

$350 = 8 + 9n$

$350 - 8 = 9n$

$9n = 342$

$n = \frac{342}{9}$

$n = 38$

$$S_n = \frac{n}{2}[a + L]$$

$$S_{38} = \frac{38}{2}[17 + 350]$$

$S_{38} = 19(367)$

$S_{38} = 6973$

7. Find the sum of first 22 terms of an AP in which d = 7 and 22nd term is 149

Ans. : $d = 7$

$a_{22} = 149$

$a + 21d = 149$

$a + 21(7) = 149$

$a + 21(7) = 149$

$a + 147 = 149$

$a = 149 - 147$

$a = 2$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{22} = \frac{22}{2}[2(2) + (22-1)(7)]$$

$S_{22} = 11\ [\ 4 + 147]$

$S_{22} = 11\ [\ 151]$

$S_{22} = 1661$

8. Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectivley

Ans.: $a_2 = 14$ $\quad a_2 = 14$

$a_3 = 18$ $\quad a + d = 14$

$d = a_3 - a_2$ $\quad a + 4 = 14$

$= 18 - 14$ $\quad a = 14 - 4$

$d = 4$ $\quad a = 10$

$S_{51} = ?$ $\quad S_n = \frac{n}{2}[2a + (n-1)d]$

$$S_{51} = \frac{51}{2}[2(10) + (51-1)(4)]$$

$$S_{51} = \frac{51}{2}[20 + (50)(4)]$$

$$S_{51} = \frac{51}{2}[20 + 200]$$

$$S_{51} = \frac{51}{2}[220]$$

$S_{51} = 51 \text{ x } 110 = 5610$

9. If the sum of first 7 terms of an AP is 49 and that of 17terms is 289. Find the sum of first n terms.

Ans. : $S_n = \frac{n}{2}[2a + (n-1)d]$ $\quad S_n = \frac{n}{2}[2a + (n-1)d]$

$S_7 = \frac{7}{2}[2a + (7-1)d]$ $\quad S_{17} = \frac{17}{2}[2a + (17-1)d]$

$49 = \frac{7}{2}[2a + 6d]$ $\quad 289 = \frac{17}{2}[2a + 16d]$

$\frac{49 \text{ x } 2}{7} = [2a + 6d]$ $\quad \frac{289 \text{ x } 2}{17} = [2a + 16d]$

$14 = 2a + 6d$.....(1) $\quad 34 = 2a + 16d$ .....(2)

Subtract Eqn.(1) from Eqn.(2)

$2a + 16d = 34$

$2a + 6d = 14$

(-) (-) (-)

$10d = 20$

$d = 2$

$2a + 6d = 14$

$2a + 6(2) = 14$

$2a + 12 = 14$

$2a = 14 - 12$

$2a = 2$

$a = 1$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_n = \frac{n}{2}[2(1) + (n-1)2]$

$S_n = \frac{n}{2}[2 + 2n - 2]$

$S_n = \frac{n}{2}[2n]$

$S_n = n^2$

10. (i) Show that $a_1, a_2, a_3 \ldots\ldots\ldots\ldots a_n$. form an AP where a_n is defined as 3 +4n & also. find the sum of thefirst 15 terms?

Ans. : $a_n = 3 + 4n$

$a_1 = 3 + 4(1) = 3 + 4 = 7$

$a_2 = 3 + 4(2) = 3 + 8 = 11$

$d = a_2 - a_1 = 11 - 7 = 4$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{15} = \frac{15}{2}[2(7) + (15-1)(4)]$

$S_{15} = \frac{15}{2}[14 + 14(4)]$

$S_{15} = \frac{15}{2}[14 + 56]$

$S_{15} = \frac{15}{2}[70]$

$S_{15} = 15 \times 35$

$S_{15} = 525$

Sum to 15 terms is 525

10. (ii)Show that $a_1, a_2, a_3 \ldots\ldots\ldots\ldots a_n$..... form an AP where a_n is defined as 9 - 5n and alose find the sum of thefirst 15 terms?

Ans. : $a_n = 9 - 5n$

$a_1 = 9 - 5(1) = 9 - 5 = 4$

$a_2 = 9 - 5(2) = 9 - 10 = -1$

$d = a_2 - a_1 = -1 - 4 = -5$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{15} = \frac{15}{2}[2(4) + (15-1)(-5)]$$

$$S_{15} = \frac{15}{2}[8 + 14(-5)]$$

$$S_{15} = \frac{15}{2}[8 - 70]$$

$$S_{15} = \frac{15}{2}[-62]$$

S_{15} = 15 x -31

S_{15} = -465

11. If the sum of the first n terms of an AP is $4n-n^2$, what is the first term? What is the sum of first two terms? What is the second term? Similarly, find the 3rd, the 10th and nth term?

Ans. : $S_n = 4n - n^2$

$S_1 = 4(1) - 1^2 = 4 - 1 = 3$

$a = S_1 = 3$

$S_2 = 4(2) - 2^2 = 8 - 4 = 4$

$S_2 = 4$

$a_1 + a_2 = 4$

$3 + a_2 = 4$

$a_2 = 4 - 3$

$a_2 = 1$

$d = a_2 - a_1 = 1 - 3 = -2$

$a_3 = a_2 + d = 1 + (-2) = -1$

$a_{10} = a + 9d$

$a_{10} = 3 + 9(-2)$

$a_{10} = 3 - 18$

$a_{10} = -15$

$a_n = a + (n-1)d$

$a_n = 3 + (n-1)(-2)$

$a_n = 3 - 2n + 2$

$a_n = 5 - 2n$

12. Find the sum of the first 40 positive integers divisible by 6

Ans. : 6 + 12 + 18 + ...

a = 6, n = 40, S_n = ?

$d = a_2 - a_1 = 12 - 6 = 6$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{40} = \frac{40}{2}[2(6) + (40-1)(6)]$$

S_{40} =20 [12 + 39(6)]

S_{40} =20 [12 + 234]

S_{40} =20 [246]

S_{40} = 4920

13. Find the sum of the first 15 multiples of 8

Ans. : 8 + 16 + 24 + ...

a = 8

d = a_2 - a_1

= 16 - 8

d = 8

n = 15

S_n = ?

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{15} = \frac{15}{2}[2(8) + (15-1)(8)]$$

$$S_{15} = \frac{15}{2}[16 + 14(8)]$$

$$S_{15} = \frac{15}{2}[16 + 112]$$

$$S_{15} = \frac{15}{2}[128]$$

S_{15} = 15 x 64 = 960

14. Find the sum of the odd numbers between 0 and 50

Ans. : 1+3+5+.....+49

a = 1

d = a_2 - a_1

= 3 - 1 = 2

n = ?

a_n = 49

S_n = ?

a_n = a + (n-1)d

49 =1 + (n-1)2

49 =1 + 2n-2

49 = 2n - 1

2n =49 +1

2n = 50

n = 25

$$S_n = \frac{n}{2}[2a + (n-1)d$$

$S_{25} = \frac{25}{2}[2(1) + (25-1)2$

$S_{25} = \frac{25}{2}[2 + 50 - 2]$

$S_{25} = \frac{25}{2}[50]$

$S_{25} = 25 \times 25$

$S_{25} = 625$

Sum of odd numbers between 0 and 50 is 625

15. A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows : Rs.200 for the first day, Rs.250 for the second day, Rs. 300 for the third day etc. the penalty for each succeeding day being Rs. 50 more than for the precceding day. How much money the contractor has to pay as penalty. if he has delayed the work for 30 days?

Ans. : Ans. : 200+250+300+.....

a = 200

d = 250 - 200

d = 50

n = 30

$S_{30} = ?$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{30} = \frac{30}{2}[2(200) + (30-1)(50)]$

$S_{30} = 15\ [400 + 29(50)]$

$S_{30} = 15\ [400 + 1450]$

$S_{30} = 15\ [1850]$

$S_{30} = 27750$

Contractor has to pay Rs. 27, 750

16. A sum of Rs. 700 is to used to give seven cash prizes to students of a school for their overall academic performance. If each prize is Rs. 20 less than its precding prize, find the value of each of the prize?

Ans.: a = ? d = -20 n = 7 $S_n = 700$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$700 = \frac{7}{2}[2a + (7-1)(-20)]$

$200 = 2a - 120$

$700 = \frac{7}{2}[2a + (6)(-20)]$

200 + 120 = 2a

$\frac{700x2}{7} = [2a - 120]$

2a = 320

a = 160

The Seven prizes are 160, 140, 120, 100, 80, 60 & 40

17. In a school,students thought of palnting trees in and around the school to reduce air pollution. It was decided that the number iof trees, that each section of each class will plant,will be the same as the class, in which they are studing e.g. a section of Class I will plant 1 tree, a section of Class II will plant 2 trees and so on till Class XII. There are three sections of each class. How many trees will be planted by the students?

Ans. : 3 [1 + 2 + 3 +.....+12]

$= 3\ S_{12}$

= 3 x 6 x 13

$= 3 \text{ x } \frac{12(12+1)}{2} = 234$

Total number of trees planted 234

18. A Spiral is made up of successive semicircles with centres alternately at A and B starting with centre at A of radii 0.5 cm, 1.0 cm. 1.5 cm, 2.0 cm .... What is the total length fo such a spiral made up of 13 consecutive semicirlces? (Take $\pi = \frac{22}{7}$)

Ans.: $0.5(\pi)+1.0(\pi)+1.5(\pi)+2.5(\pi) + ... + 6.5(\pi)$

$= (\pi)(0.5)\ (1 + 2 + 3 + ...+13)$

a = 1 $\qquad S_n = \frac{n}{2}[2a + (n-1)d]$

d = 2 - 1 = 1 $\qquad S_{13} = \frac{13}{2}[2(1) + (13-1)(1)]$

n = 13 $\qquad S_{13} = \frac{13}{2}[2 + 12]$

S_{13} = ? $\qquad S_{13} = \frac{13}{2}[14]$

S_{13} = 13 x 7 = 91

The total length fo such a spiral made up of 13 consecutive semicirlces is

$= \frac{22}{7} \text{ x } \frac{1}{2} \text{ x } S_{13} \quad = \frac{11}{7} \text{ x } 91 = 11 \text{ x } 13 = 143$

19. 200 logs are stacked in the floowing manner : 20 logs in the botton row.19 in the next row,18 in the row next to it and so on. In how many rows are the 200 logs placed and how many logs are in the top row.?

Ans. : a = 20

d = -1

n = ?

$S_n = 200$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$200 = \frac{n}{2}[2(20) + (n-1)(-1)$

200 x 2 = n [40-n+1]

400 = n[-n+41]

$400 = -n^2 + 41n$

$n^2 - 41n + 400 = 0$

$n^2 - 16n - 25n + 400 = 0$

n(n-16) - 25(n-16) = 0

(n - 16) (n - 25) = 0

n - 16 = 0 or n - 25 = 0

n = 16 or n = 25

If n = 16

$a_{16} = a + 15d$

$a_n = 20 + 15(-1)$

$a_n = 20 - 15 = 5$

Therefore number of rows is 16 and first row contains 5 logs