Bismihi Tala

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9^{th} Standard
MATHEMATICS

Chapters :

NUMBER SYSTEM

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NUMBER SYSTEM

1) **Number :** A number is a figure or a symbol used to represent the quantity

Eg. : 5, 5, ೫, ۵ etc

2) **Positive numbers** : Numbers greater than zero (0) are called positive numbers

Eg. : 7, +9, 0.3, 23 etc

3) Negative numbers : Numbers Less than zero (0) are called negative numbers

Eg. : -3, -2.3, - 23 etc

4) Even numbers : The numbers which are divisible by 2 are called even numbers

Eg. : 6, 14, 2022, 1946 etc

5) Odd numbers : The numbers which are not divisible by 2 are called odd numbers

Eg. : 15, 37, 135 2047 etc

6) Prime numbers : The numbers which are divisible by 1 or itself are called prime numbers numbers.   Eg. : 2, 3, 5, 7, 11, 13, 17, 23 etc

7) Triangular numbers : The triangular number sequence is the representation of the numbers in the form of equilateral triangle arranged in a series or sequence. These numbers are in a sequence of 1, 3, 6, 10, 15, 21, 28,36, 45, and so on.

8) Composite numbers : composite numbers are the numbers which have more than two factors, unlike prime numbers which have only two factors, i.e. 1 and the number itself. These numbers are also called composites.

All the natural numbers which are not prime numbers are composite numbers as they can be divided by more than two numbers. For example, 6 is composite because it is divisible by 1, 2, 3 and even 6, such as

$6÷2 = 3$, $6÷3 = 2$, $6÷6 = 1$, $6÷1 = 6$

Types of Numbers

1. Natural numbers : Counting numbes are called natural numbers

Eg. : $N = \{1, 2, 3, 4, \ldots\ldots\ldots\ldots \infty \text{(infinity)}\}$

Properties of Natural numbers

1) Natural number starts from 1 to ∞ (infinity)

2) All Natural numbers are postive

3) The difference between any two consecutive natural numbers is '1'

4) The set of Natural numbers is denoted by N

1) Closure Property

(a) Addition

For all $a, b \in N$, $a+b \in N$

Let $a = 12 \in N$, $b = 34 \in N$

$a + b = 12 + 34 = 46 \in N$

Therefore Natural Number is closed under addition

(b) Multiplication

For all $a, b \in N$, $a \times b \in N$

Let $a = 10 \in N$, $b = 15 \in N$

$a \times b = 10 \times 15 = 150 \in N$

Therefore Natural Number is closed under Multiplication

(c) Subtraction

Let $a = 5 \in N$, $b = 13 \in N$

$a - b = 5 - 13 = -8 \notin N$

Therefore Natural Number is not closed under Subtraction

(d) Division

Let $a = 8 \in N$, $b = 5 \in N$

$a \div b = 8 \div 5 = \frac{8}{5} \notin N$

Therefore Natural Number is not closed under Subtraction

6) Commutative Property

(a) Addition

For all $a, b \in N$, $a + b = b + a \in N$

Let $a = 7 \in N$, $b = 18 \in N$

$a + b = 7 + 18 = 25 \in N$

$b + a = 18 + 7 = 25 \in N$

Therefore $a + b = b + a \in N$

Natural Number is Commutative under addition

(b) Multiplication

For all $a, b \in N$, $a \times b = b \times a \in N$

Let $a = 11, b = 12 \in N$

$a \times b = 11 \times 12 = 132 \in N$

$b \times a = 12 \times 11 = 132 \in N$

Therefore $a \times b = b \times a \in N$

Natural Number is Commutative under Multiplication

(c) Subtraction

Let $a = 6, b = 19 \in N$

$a - b = 6 - 19 = -13 \notin N$

$b - a = 19 - 6 = 13 \in N$

Therefore $a - b \neq b - a$

Natural Number is not Commutative under Subtraction

(d) Division

Let $a = 6, b = 7 \in N$

$a \div b = 6 \div 7 = \frac{6}{7} \notin N$

$b \div a = 7 \div 6 = \frac{7}{6} \notin N$

Therefore $a \div b \neq b \div a$

Natural Number is not Commutative under Division

2) Assosiative Property

(a) Addition

For all $a, b, c \in N$, $a + (b + c) = (a + b) + c \in N$

Let $a = 3 \in N$, $b = 4$, $c = 5 \in N$

$a + (b + c) = 3 + (4 + 5) = 3 + 9 = 12 \in N$

$(a + b) + c = (3 + 4) + 5 = 7 + 5 = 12 \in N$

$a + (b + c) = (a + b) + c$

Therefore Natural Number is Associative under addition

(b) Multiplication

Let $a = 5, b = 6, c = 7 \in N$

$a \times (b \times c) = 5 \times (6 \times 7) = 5 \times 42 = 210 \in N$

$(a \times b) \times c = (5 \times 6) \times 7 = 30 \times 7 = 210 \in N$

$a \times (b \times c) = (a \times b) \times c$

Therefore Natural Number is Associative under Multiplication

(c) Subtraction

Let $a = 2 \in N, b = 8, c = 4 \in N$

$a - (b - c) = 2 - (8 - 4) = 2 - 4 = -2 \notin N$

$(a - b) - c = (2 - 8) - 4 = -6 - 4 = -10 \notin N$

$a - (b - c) \neq (a - b) - c$

Therefore Natural Number is not Associative under Subtraction

(c) Division

Let $a = 2 \in N, b = 8, c = 4 \in N$

$a \div (b \div c) = 2 \div (8 \div 4) = 2 \div \frac{8}{4} = 2 \div 2 = 1 \in N$

$(a \div b) \div c = (2 \div 8) \div 4 = \frac{2}{8} \div 4 = \frac{2}{8} \times \frac{1}{4} = \frac{1}{16} \notin N$

$a \div (b \div c) \neq (a \div b) \div c$

Therefore Natural Number is not Associative under Division

3) Additive Identity

For all $a \in N, a + 0 = 0 + a = a \in N$

'0' is called Multiplictive Identity of Natural numbers

Example Let $a = 3 \in N$

$3 + 0 = 0 + 3 = 3$

3) Multiplicative Identity

For all $a \in N, a \times 1 = 1 \times a = a \in N$

'1' is called Multiplictive Identity of Natural numbers

Example Let $a = 3 \in N$

$3 \times 1 = 1 \times 3 = 3$

2 Whole numbers : Natural numbers including zero(0) are called Whole numbers

Eg. : W = { 0, 1, 2, 3, 4, ... ∞ (infinity)]

Note : The set of Whole numbers is denoted by W

Properties of Whole numbers

1) Closure Property

(a) Addition

Let $a = 12, b = 14 \in W$

$a + b = 12 + 14 = 26 \in W$

Therefore Whole number is closed under addition

(b) Multiplication

Let $a = 7 \in N, b = 0 \in W$

$a \times b = 7 \times 0 = 0 \in W$

Therefore Whole number is closed under Multiplication

(c) Subtraction

Let $a = 10, b = 13 \in W$

$a - b = 10 - 13 = -3 \notin W$

Therefore Whole number is not closed under Subtraction

(d) Division

Let $a = 8, b = 5 \in W$

$a \div b = 8 \div 5 = \frac{8}{5} \notin W$

Therefore Whole number is not closed under Subtraction

2) Assosiative Property

(a) Addition

Let $a = 3, b = 4, c = 5 \in W$

$a + (b + c) = 3 + (4 + 5) = 3 + 9 = 12 \in W$

$(a + b) + c = (3 + 4) + 5 = 7 + 5 = 12 \in W$

$a + (b + c) = (a + b) + c$

Therefore Whole number is Assosiative under addition

(b) Multiplication

Let $a = 5, b = 6, c = 7 \in W$

$a \times (b \times c) = 5 \times (6 \times 7) = 3 \times 42 = 210 \in W$

$(a \times b) \times c = (5 \times 6) \times 7 = 30 \times 7 = 210 \in W$

$a \times (b \times c) = (a \times b) \times c$

Therefore Whole number is Assosiative under Multiplication

(c) Subtraction

Let $a = 2, b = 8, c = 4 \in W$

$a - (b - c) = 2 - (8 - 4) = 2 - 4 = -2 \notin W$

$(a - b) - c = (2 - 8) - 4 = -6 - 4 = -10 \notin W$

$a - (b - c) \neq (a - b) - c$

Therefore Whole number is not Assosiative under Subtraction

(c) Division

Let $a = 2, b = 8, c = 4 \in W$

$a \div (b \div c) = 2 \div (8 \div 4) = 2 \div \frac{8}{4} = 2 \div 2 = 1 \in W$

$(a \div b) \div c = (2 \div 8) \div 4 = \frac{2}{8} \div 4 = \frac{2}{8} \times \frac{1}{4} = \frac{1}{16} \notin W$

$a \div (b \div c) \neq (a \div b) \div c$

Therefore Whole number is not Assosiative under Division

3) Additive Identity

For all $a \in W$, $a + 0 = 0 + a = a \in W$

'0' is called additive Identity of Whole numbers

Example Let $a = 7 \in W$

$7 + 0 = 0 + 7 = 7$

4) Multiplicative Identity

For all $a \in W$, $a \times 1 = 1 \times a = a \in W$

'1' is called Multiplictive Identity of Whole numbers

Example Let $a = 5 \in W$

$5 \times 1 = 1 \times 5 = 5$

3. Integers : Integers Contain both positive numbers, negative numbers along with zer(0)

Eg. : Z= { ....-4, -3, -2, -1,0, 1, 2, 3, 4, ... }

Properties of Integers

1) The set of Integers contain positive numbers, negative numbers along with zer(0)

2) It does not contain fractions

3) The set of Integers is denoted by Z

1) Closure Property

(a) Addition

Let $a = -12, b = 14 \in Z$

$a + b = -12 + 14 = 2 \in Z$

Therefore Integers is closed under addition

(b) Multiplication

Let $a = -7, b = 3 \in Z$

$a \times b = -7 \times 3 = -21 \in Z$

Therefore Integers is closed under Multiplication

(c) Subtraction

Let $a = -10 \in Z, b = 13 \in Z$

$a - b = -10 - 13 = -23 \in Z$

Therefore Integers is closed under Subtraction

(d) Division

Let $a = 8 \in N, b = -5 \in Z$

$a \div b = 8 \div -5 = -\frac{8}{5} \notin Z$

Therefore Integers is not closed under Subtraction

2) Assosiative Property

(a) Addition

Let $a = -3 \in Z, b = 4, c = 0 \in Z$

$a + (b + c) = -3 + (4 + 0) = -3 + 4 = 1 \in Z$

$(a + b) + c = (-3 + 4) + 0 = 1 + 0 = 1 \in Z$

$a + (b + c) = (a + b) + c$

Therefore Integers is Associative under addition

(b) Multiplication

Let $a = -5 \in Z, b = 6, c = -7 \in Z$

$a \times (b \times c) = -5 \times (6 \times -7) = -5 \times -42 = 210 \in Z$

$(a \times b) \times c = (-5 \times 6) \times -7 = -30 \times -7 = 210 \in Z$

$a \times (b \times c) = (a \times b) \times c$

Therefore Integers is Associative under Multiplication

(c) Subtraction

Let $a = 2 \in Z, b = -8, c = 4 \in Z$

$a - (b - c) = 2 - (-8 - 4) = 2 - (-12) = 2+12 = 14 \in Z$

$(a - b) - c = [2 - (-8)] - 4 = 10 - 4 = 6 \in Z$

$a - (b - c) \neq (a - b) - c$

Therefore Integers is not Associativeunder Subtraction

(c) Division

Let $a = 2, b = 8, c = -4 \in Z$

$a \div (b \div c) = 2 \div (8 \div -4) = 2 \div -\frac{8}{4} = 2 \div -2 = -1 \in Z$

$(a \div b) \div c = (2 \div 8) \div -4 = \frac{2}{8} \div -4 \quad \frac{1}{4} \text{ x } -\frac{1}{4} = -\frac{1}{16} \notin Z$

$a \div (b \div c) \neq (a \div b) \div c$

Therefore Integers is not Associative under Division

3) Additive Identity

For all $a \in Z$, $a + 0 = 0 + a = a \in Z$

'0' is called additive Identity of Whole numbers

Example Let $a = 7 \in Z$

$7 + 0 = 0 + 7 = 7$

4) Multiplicative Identity

For all $a \in Z$, $a \text{ x } 1 = 1 \text{ x } a = a \in Z$

'1' is called Multiplictive Identity of Whole numbers

Example Let $a = 5 \in Z$

$5 \text{ x } 1 = 1 \text{ x } 5 = 5$

5) Additive inverse

For all $a \in Z$, $a + (-a) = 0$

'-a' is called additive inverse of a in integers

Example 1) Let $a = 7 \in Z$

There fore -7 is called the additive inverse of 7

2) Let $a = -10 \in Z$

There fore +10 is called the additive inverse of -10

4) Rational Number : A number which can be written in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$ is caled a rational number

Eg. : $\frac{2}{7}, -\frac{5}{9}, 0,$ etc

Properties of Rational numbers:

1) The set of Rational numberscontain positive numbers,negative numbers,fractions along with zero

2) The set of rational numbers is denoted by Q

1) Closure Property

(a) Addition

Let $a = 12, b = \frac{7}{6} \in Q$

$a + b = 12 + \frac{7}{6} = \frac{12 \text{ x } 6 + 7}{6} = \frac{72 + 7}{6} = \frac{79}{6} \in Q$

Therefore Rational numbes is closed under addition

(b) Multiplication

Let $a = -7, b = \frac{3}{2} \in Q$

$a \text{ x } b = -7 \text{ x } \frac{3}{2} = \frac{-21}{2} \in Q$

Therefore Rational numbes is closed under Multiplication

(c) Subtraction

Let $a = \frac{7}{4}, b = \frac{8}{4} \in Q$

$a - b = \frac{7}{4} - \frac{8}{4} = \frac{7-8}{4} = \frac{-1}{4} \in Q$

Therefore Rational numbes is closed under Subtraction

(d) Division

Let $a = 3, b = 0 \in Q$

$a \div b = 3 \div 0 = \frac{3}{0} = \text{undefined} \notin Q$

Therefore Rational numbes is not closed under Subtraction

2) Assosiative Property

(a) Addition

Let $a = 6 \in Z, b = -15, c = \frac{1}{2} \in Q$

$a + (b + c) = 6 + (-15 + \frac{1}{2}) = 6 + \frac{-30+1}{2} = 6 + \frac{-29}{2} = \frac{12-29}{2} = \frac{-17}{2} \in Q$

$(a + b) + c = (6 - 15) + \frac{1}{2} = -9 + \frac{1}{2} = \frac{-18+1}{2} = \frac{-17}{2} \in Q$

$a + (b + c) = (a + b) + c$

Therefore Rational numbes is Associative under addition

(b) Multiplication

Let $a = \frac{15}{2}, b = -\frac{3}{15}, c = 9 \in Q$

$\frac{15}{2} \times (-\frac{3}{15} \times 9) = \frac{15}{2} \times -\frac{9}{5} = -\frac{27}{2} \in Q$

$(a \times b) \times c = (\frac{15}{2} \times -\frac{3}{15}) \times 9 = -\frac{3}{2} \times 9 = -\frac{27}{2} \in Q$

a x (b x c) = (a x b) x c

Therefore Rational numbers is Associative under Multiplication

(c) Subtraction

Let $a = 2 \in Q$, $b = -8$, $c = 4 \in Q$

$a - (b - c) = 2 - (-8 - 4) = 2 - (-12) = 2+12 = 14 \in Q$

$(a - b) - c = [2 - (-8)] - 4 = 10 - 4 = 6 \in Q$

$a - (b - c) \neq (a - b) - c$

Therefore Rational numbers is not Associativeunder Subtraction

3) Additive Identity

For all $a \in Q$, $a + 0 = 0 + a = a \in Q$

'0' is called additive Identity of Whole numbers

Example Let $a = \frac{15}{14} \in Q$

$\frac{15}{14} + 0 = 0 + \frac{15}{14} = \frac{15}{14}$

4) Multiplicative Identity

For all $a \in Q$, $a \times 1 = 1 \times a = a \in Q$

'1' is called Multiplictive Identity of Whole numbers

Example Let $a = -\frac{7}{5} \in Q$

$-\frac{7}{5} \times 1 = 1 \times -\frac{7}{5} = -\frac{7}{5}$

5) Additive inverse

For all $a \in Q$, $a + (-a) = 0$

'-a' is called additive inverse of a in integers

Example 1) Let $a = \frac{1}{2} \in Q$

There fore $-\frac{1}{2}$ is called the additive inverse of 7

2) Let a $= -10 \in Q$

There fore +10 is called the additive inverse of -10

6) Multiplicative inverse

For all $a \in Q$, $a \times (\frac{1}{a}) = 1$

'-a' is called additive inverse of a in integers

Example 1) Let $a = \frac{1}{2} \in Q$

There fore 2 is called the Multiplicative inverse of $\frac{1}{2}$

2) Let $a = -\frac{7}{5} \in Q$

There fore $-\frac{5}{7}$ is called the additive inverse of $-\frac{7}{5}$

Exercise 1.1

1. Is Zero a rational number? Can you write it in the form $\frac{p}{q}$ where p and q are integers & $q \neq 0$

Soln. : Zero (0) is a rational number

because $0 = \frac{0}{1}$ where 0 & 1 are integers and $1 \neq 0$

2) Find 6 rationa numbers between 3 and 4

Soln. : $3 = \frac{3 \times 10}{1 \times 10} = \frac{30}{10}$

$4 = \frac{4 \times 10}{1 \times 10} = \frac{40}{10}$

Ten Rational numbers between $\frac{30}{10}$ and $\frac{40}{10}$ are

$\frac{31}{10}, \frac{32}{10}, \frac{33}{10}, \frac{34}{10}, \frac{35}{10}$ and $\frac{36}{10}$

$= \frac{31}{10}, \frac{16}{5}, \frac{33}{10}, \frac{17}{5}, \frac{7}{2}$ and $\frac{18}{5}$

3) Find 5 rational numbes between $\frac{3}{5}$ and $\frac{4}{5}$

Soln. : The given rational numbers are $\frac{2}{3}$ and $\frac{4}{5}$

$$\frac{3}{5} = \frac{3 \text{ x } 6}{5 \text{ x } 6} = \frac{18}{30}$$

$$\frac{4}{5} = \frac{4 \text{ x } 6}{5 \text{ x } 6} = \frac{24}{30}$$

5 Rational numbers between $\frac{18}{30}$ and $\frac{24}{30}$ are

$\frac{19}{30}, \frac{20}{30}, \frac{31}{30}, \frac{32}{30}$ and $\frac{33}{30}$

$= \quad \frac{19}{30}, \frac{2}{3}, \frac{31}{30}, \frac{16}{15}$ and $\frac{33}{30}$

4) State whether the following statements are true or false. Give reasons for your answers

(i) Every natural number is a whole number

Soln.: True

(Because Set of Natural numbers is a sub set of whole number set)

(ii) Every integers is a whole number

Soln.: False

(Because negative integers does not belongs to set of whole number)

(iii) Every rational number is a whole number

Soln. : False

(Because negative integers, and fractions does not belongs to set of rational number)

Irrational numbers : Irrational numbers are the real numbers that cannot be represented as a simple fraction. It cannot be expressed in the form of a ratio, such as $\frac{p}{q}$, where p and q are integers, $q \neq 0$ It is a contradiction of rational numbers.

Examples : $\sqrt{2}, \sqrt{3}, \sqrt{15}, \pi\pi 0.101101101110.....$

Real numbers : Real numbers are simply the combination of rational and irrational numbers, in the number system.

Exercise 1.2

1) State whether the following statements are True or False. Justify your answers

(i) Every irrational number is a real number

Soln. : True

(Because Set of Irrational numbers is a sub set of Real number set)

(ii) Every point on the number line is of the form $\sqrt{m}$, where m is a natural number

Soln. : False

(Because negative numbers cannot be the square root of any natural number.)

(iii) Every Real number is a irrational number

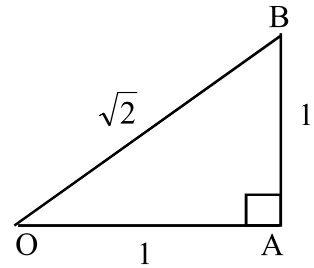
Soln. : False

(Because Rational numbers does not belongs to set of irrational number)

2) Are the square roots of all positive itegers irrational? if not, give an example of te square root of a number that is a rational number

Soln. :No, if we take a positive integer, say 9, its square root is 3, which is a rational number.

3) Locate $\sqrt{2}$ on the number line



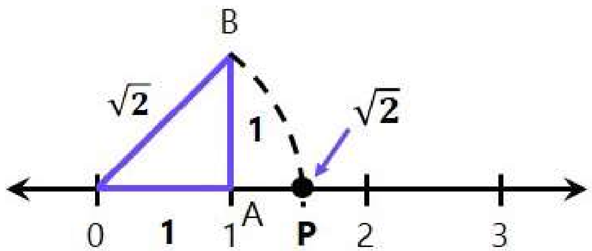
In $\Delta OAB, \angle A = 90^0$

$OB^2 = OA^2 + AB^2$ (Pythagoras Theorem)

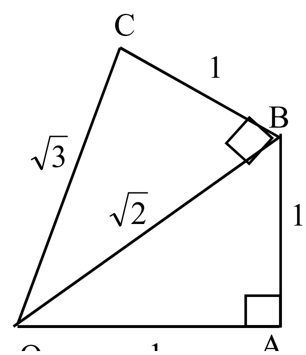
$OB^2 = 1^2 + 1^2$

$OB^2 = 2$

$OB = \sqrt{2}$



4) Locate $\sqrt{3}$ on the number line



In $\Delta OAB, \angle A = 90^0$

$OB^2 = OA^2 + AB^2$ (Pythagoras Theorem)

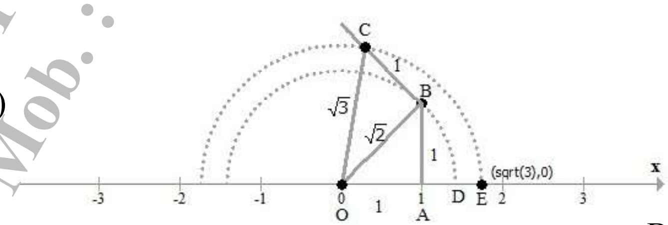
$OB^2 = 1^2 + 1^2$

$OB^2 = 2$

$OB = \sqrt{2}$

In $\Delta OBC, \angle B = 90^0$

$OC^2 = OB^2 + BC^2$ (Pythagoras Theorem)

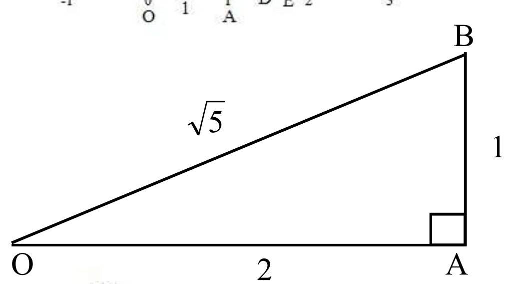


$OC^2 = \sqrt{2}^{\,2} + 1^2$

$OC^2 = 3$

$OC = \sqrt{3}$

5) Show how $\sqrt{5}$ can be represented on the number line



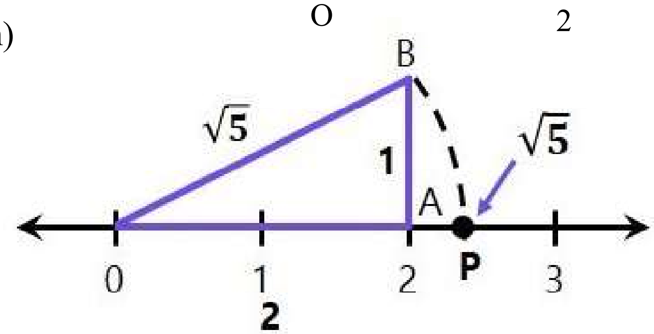
In $\Delta OAB, \angle A = 90^0$

$OB^2 = OA^2 + AB^2$ (Pythagoras Theorem)

$OB^2 = 2^2 + 1^2$

$OB^2 = 5$

$OB = \sqrt{5}$



Terminating decimals : A terminating decimal is a decimal, that has an end digit. It is a decimal, which has a finite number of digits(or terms). Example: 0.15, 0.86, etc

Non Terminating decimals : Non-terminating decimals are the one that does not have an end term. It has an infinite number of terms.

Non Terminating recurring decimals : A repeating decimal or recurring decimal is decimal representation of a number whose digits are periodic (repeating its values at regular intervals) and the infinitely repeated portion is not zero. ... The infinitely repeated digit sequence is called the repetend or reptend.

Exercise 1.3

1. Write the following in decimal form and say what kind of decimal expansion each has.

(i) $\frac{36}{100} = 0.36$

Soln. : Therefore the decimal expansion is Terminating

(ii) $\frac{1}{11}$

11) 100 (090909..................
99
100
99
100
99
1

$\frac{1}{11}$ = 090909.................. = $0.\overline{09}$

Soln. : Therefore the decimal expansion of $\frac{1}{11}$ is Non Terminating Repeating

(iii) $4\frac{1}{8} = \frac{33}{8}$

8) 33 (4.125
32
10
8
20
16
40
40
00

$4\frac{1}{8} = \frac{33}{8} = 4.125$

Soln. : Therefore the decimal expansion of $4\frac{1}{8}$ is Terminating

(iv) $\frac{3}{13}$

$$
\begin{array}{r|l}
13) & 30\ (0.230769 \\
 & \underline{26} \\
 & 40 \\
 & \underline{39} \\
 & 100 \\
 & \underline{91} \\
 & 90 \\
 & \underline{78} \\
 & 120 \\
 & \underline{117} \\
 & 3
\end{array}
$$

$\frac{3}{13} = 0.\overline{230769}$

Therefore the decimal expansion of $\frac{3}{13}$ is Non Terminating Repeating

(v) $\frac{2}{11}$

$$
\begin{array}{r|l}
11) & 20\ (0.18 \\
 & \underline{11} \\
 & 90 \\
 & \underline{88} \\
 & 20
\end{array}
$$

$\frac{2}{11} = 0.\overline{18}$

Therefore the decimal expansion of $\frac{2}{11}$ is Non Terminating Repeating

(vI) $\frac{329}{400}$

Soln. :

$$
\begin{array}{r|l}
400) & 3290\ (0.8225 \\
 & \underline{3200} \\
 & 900 \\
 & \underline{800} \\
 & 1000 \\
 & \underline{800} \\
 & 2000 \\
 & \underline{2000} \\
 & 0000
\end{array}
$$

$$\frac{329}{400} = 0.8225$$

Therefore the decimal expansion of $\frac{329}{400}$ is Terminating

2. You know that $\frac{1}{7} = 0.\overline{142857}$. Can you predict what the decimal expansions of $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}$ are, without actually doing the long division? If so, how?

Soln. : $\frac{1}{7} = 0.\overline{142857}$

$$2 \text{ x } \frac{1}{7} = 2 \text{ x } 0.\overline{142857} = 0.\overline{285714}$$

$$3 \text{ x } \frac{1}{7} = 3 \text{ x } 0.\overline{142857} = 0.\overline{428571}$$

$$4 \text{ x } \frac{1}{7} = 4 \text{ x } 0.\overline{142857} = 0.\overline{571428}$$

$$5 \text{ x } \frac{1}{7} = 5 \text{ x } 0.\overline{142857} = 0.\overline{714285}$$

$$6 \text{ x } \frac{1}{7} = 6 \text{ x } 0.\overline{142857} = 0.\overline{857142}$$

3. Express the following in the $\frac{p}{q}$. where p and q are integers and $q \neq 0$

i) $0.\overline{6}$

Soln. : Let $x = 0.\overline{6}$

$x = 0.666666$....................

Multiply by 10 on both sides

$10x = 6.66666$....................

$10x = 6+0.66666$....................

$10x = 6+ x$

$10x - x = 6$

$9x = 6$

$$x = \frac{6}{9} = \frac{2}{3}$$

ii) $0.4\overline{7}$

Soln. : Let $x = 0.4\overline{7}$

$x = 0.47777777777$.................... (1)

Multiply by 10 on both sides

$10x = 4.7777777$ ....(2)

Subtract Eq(1) from Eq(2)

$10x = 4.7777777$

$x = 0.47777777777$

$9x = 4.30 = 4.3$

$9x = \frac{43}{10}$

$x = \frac{43}{9 \times 10}$

$x = \frac{43}{90}$

iii) $0.\overline{001}$

Soln. : Let $x = 0.\overline{001}$

$x = 0.001001001$..................... (1)

Multiply by 1000 on both sides

$1000x = 1.001001$..............(2)

Subtract Eq(1) from Eq(2)

$$\begin{array}{r} 1000x = 1.001001001 \\ x = 0.001001001 \\ \hline 999x = 1.0 \end{array}$$

$999x = 1$

$x = \frac{1}{999}$

iv) $0.\overline{001}$

Soln. : Let $x = 0.\overline{001}$

$x = 0.001001001$..................... (1)

Multiply by 1000 on both sides

$1000x = 1.001001$..............(2)

Subtract Eq(1) from Eq(2)

$1000x = 1.001001001$

$x = 0.001001001$

$999x = 1.0$

$999x = 1$

$x = \frac{1}{999}$

4. Express 0.99999........... in the form $\frac{p}{q}$.

Soln. : Let $x = 0.99999$.................... (1)

Multiply by 10 on both sides

$10x = 9.99999............(2)$

$10x - x = 9.99999 - 0.99999$

$9x = 9$

$x = \frac{9}{9}$

$x = 1$

5. What can the maximum number of digits be in the repeating block of digits in the decimal expansion of $\frac{1}{17}$? Perform the division to check your answer

Soln. : $\frac{1}{17}$ 17)100 (0.0588235294117647

```
85
150
136
 40
 34
 60
 51
 90
 85
 50
 34
160
153
 70
 68
 20
 17
 30
 17
130
119
110
102
 80
 68
120
119
100
```

$\frac{1}{17} = 0.\overline{0588235294117647}$

There are 16 digits in the repeating of the decimal expansion of $\frac{1}{17}$

6. Look at several examples of rational numbers in the form $\frac{p}{q}$ $(q \neq 0)$ where p and q are integers with no common factors other than 1 and having terminating decimals representation (expansion). Can you guess what property q must satisfy?

Soln.: We observe that when q is 2, 4, 5, 8, 10............. then the decimal expansion is terminating.

For

$\frac{1}{2} = 0.5$ derminting decimal $q = 2^1$

$\frac{7}{8} = 0.875$ derminting decimal $q = 2^3$

$\frac{4}{5} = 0.8$ derminting decimal $q = 5^1$

We can also observe that the terminating decimal may be obtained in the situatation where prime factorisation of the denominator of the given fractions has the power of only 2 or only 5 or both

7. Write three numbers whose decimals expansions are non-terminating non -recurring .

Soln. : We know that all irrational numbers are non-terminating non -recurring .

1) $\pi = \frac{22}{7} = 3.1416......$

2) $\sqrt{2} = 1.414......$

3) $\sqrt{3} = 1.73205080......$

8. Find three different irrational numbers between the rational numbers $\frac{5}{7}$ and $\frac{9}{11}$

Soln. : $\frac{5}{7} = 0.\overline{714285}$

$\frac{9}{11} = 0.\overline{81}$

Three different irrational numbers between the rational numbers $\frac{5}{7}$ and $\frac{9}{11}$ are

1) 0.7234596................

2) 0.7425735................

3) 0.78123957

9. classify the following numbers as rational or irrational

i) $\sqrt{225}$

$\sqrt{225} = 15$

Therefore $\sqrt{225}$ is a rational number

ii) $\sqrt{23} = 4.79583152331$...............

$\sqrt{23}$ is a non terminating non recurring

Therefore $\sqrt{23}$ is an irrational number

iii) 0.3796

The given number is terminating

Therefore it is a rational number

iv) 7.478478....

The given number is non terminating but recurring

Therefore it is a rational number

v) 1.101001000100001............

The given number is non repeating (non recurring)

Therefore it is a irrational number

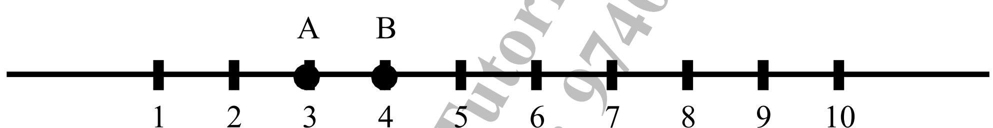
Successive Magnification

The process of visualisation is representation of real numbers on the number lien through magnifying glass is known as successive magnification

Excercise 1.4

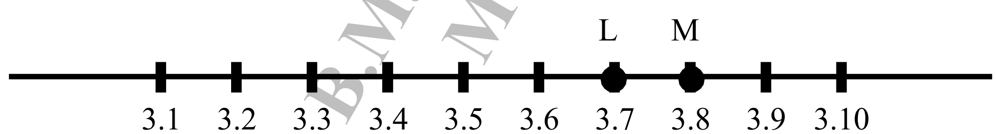
1) Visualise 3.765 on the number line

Soln. : Step 1 : The given number lies between 3 and 4



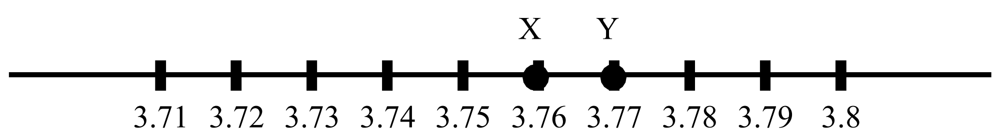
Step 2 : Magnify the interval between 3 and 4 and divide it into 10 equal parts

Step 3 : The given numbers lies between 3.7 and 3.8



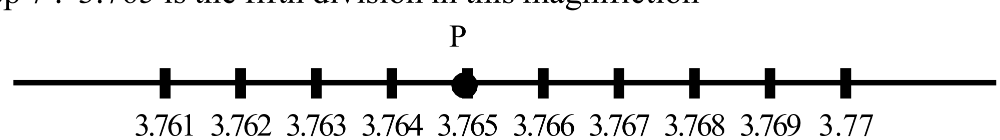
Step 4 : Divide the interval between 3.7 and 3.8 and divide into 10 equl parts and magnify it

Step 5 : The given number lies between 3.76 and 3.77

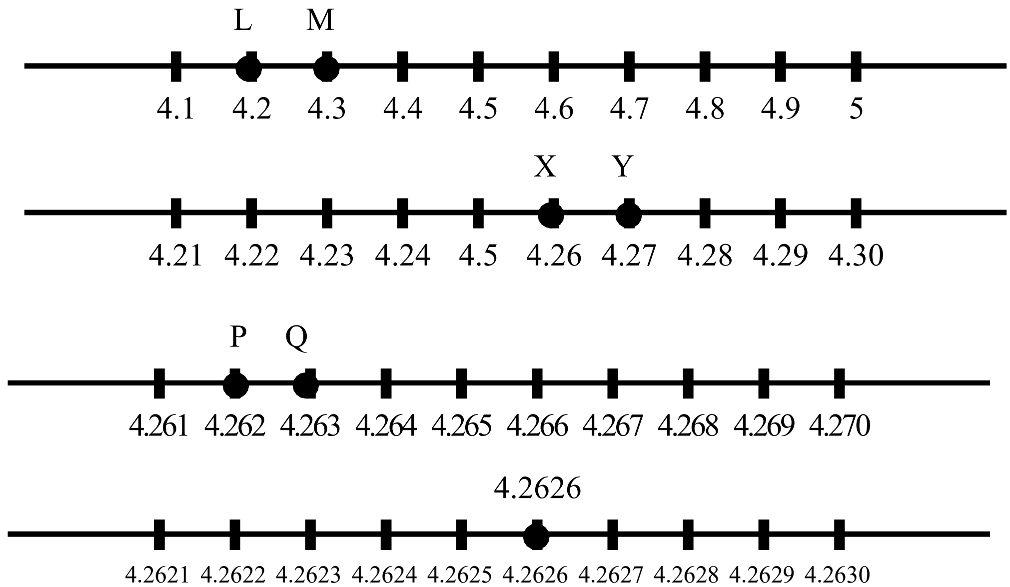


Step 6 : Magnify the interval between 3.76 and 3.77 and divide into 10 equl parts

Step 7 : 3.765 is the fifth division in this magnifiction



2) Visualise $4.\overline{26}$ on the number line



Rationalisation :

The process of multiplying an iration number with the same number is called rationalisation

Eg. : $\frac{3}{\sqrt{2}}$

in this example $\sqrt{2}$ is an irrational number to make it a rational number we multiply it with the same number i.e. $\sqrt{2}$

Since we have to balance the fraction we multiply bot numertor as well as denominator by $\sqrt{2}$

$$\frac{3}{\sqrt{2}} = \frac{3}{\sqrt{2}} \text{ x } \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$$

Laws of Indicies (Exponents/Powers)

$\forall$ a, b, m, n $\in$ R

(i) $a^m \text{ x } a^n = a^{m+n}$

(ii) $\frac{a^m}{a^n} = a^{m-n}$

(iii) $(a^m)^n = a^{mn}$

(iv) $(ab)^m = a^m \text{ x } b^m$

(v) $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

(vi) $a^0 = 1$

(vii) $a^{-n} = \frac{1}{a^n}$

Excercise 1.5

1. Classify the following numbers as rational or irrational

(i) $2-\sqrt{5}$ irrational number

(ii) $\left(3+\sqrt{23}\right)-\sqrt{23}$ rational number

Because $3+\sqrt{23}-\sqrt{23}=3$

(iii) $\frac{2\sqrt{7}}{7\sqrt{7}}$ rational number

Because $\frac{2\sqrt{7}}{7\sqrt{7}}=\frac{2}{7}$

(iv) $\frac{1}{\sqrt{2}}$ irrational number

(iv) $2\ \pi$ irrational number

2. Simplify each of the following expression

(i) $\left(3+\sqrt{3}\right)\left(2+\sqrt{2}\right)$

$= 3\left(2+\sqrt{2}\right)+\sqrt{3}.\left(2+\sqrt{2}\right)$

$= 6+3\sqrt{2}+2\sqrt{3}+\sqrt{6}$

(ii) $\left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right)$ $(a+b)(a-b)=a^2-b^2$

$= 3^2 - .\sqrt{3}^2$

$= 9- 3$

$= 6$

(iii) $\left(\sqrt{5}+\sqrt{2}\right)^2$ $(a+b)^2=a^2+b^2+2ab$

$= \sqrt{5}^2+\sqrt{2}^2+2(\sqrt{5})(\sqrt{2})$

$= 5+2+2\sqrt{10}$

$= 7+2\sqrt{10}$

(iv) $\left(\sqrt{5}-\sqrt{2}\right)\left(\sqrt{5}+\sqrt{2}\right)$ $(a+b)(a-b)=a^2-b^2$

$= \sqrt{5}^2-\sqrt{2}^2$

$= 5 - 2$

$= 3$

4. Represent $\sqrt{9.3}$ on the number line

Solution:

Step-1: Draw a line segment AB = 9.3 units and extend it to C such that BC = 1 unit.

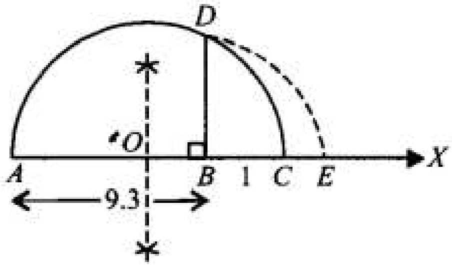
Step-2: Find mid point of AC and mark it as O.

Step-3: Draw a semicircle taking O as centre and AO as radius.

Step-4: Draw BD $\perp$ AC.

Step-5: Draw an arc taking B as centre and BD as radius meeting AC produced at E such that

BE = BD = $\sqrt{9.3}$ units



5. Rationalise the denominator of the following

(i) $\frac{1}{\sqrt{7}}$

Rationalising factor of $\sqrt{7}$ is $\sqrt{7}$

Multiply and Divide $\frac{1}{\sqrt{7}}$ by $\sqrt{7}$

$\frac{1}{\sqrt{7}} \text{ x} \frac{\sqrt{7}}{\sqrt{7}}$

$= \frac{\sqrt{7}}{(\sqrt{7})^2}$

$= \frac{\sqrt{7}}{7}$

(ii) $\frac{1}{\sqrt{7}-\sqrt{6}}$

Conjugate of $\sqrt{7}-\sqrt{6}$ is $\sqrt{7}+\sqrt{6}$

Multiply and Divide $\frac{1}{\sqrt{7}-\sqrt{6}}$ by $\sqrt{7}+\sqrt{6}$

$\frac{1}{\sqrt{7}-\sqrt{6}} \text{ x} \frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}}$

$= \frac{\sqrt{7}+\sqrt{6}}{(\sqrt{7})^2-(\sqrt{6})^2}$

$= \frac{\sqrt{7}+\sqrt{6}}{7-6}$

$= \frac{\sqrt{7}+\sqrt{6}}{1}$

$= \sqrt{7}+\sqrt{6}$

(iii) $\frac{1}{\sqrt{5}+\sqrt{2}}$

Conjugate of $\sqrt{5}+\sqrt{2}$ is $\sqrt{5}-\sqrt{2}$

Multiply and Divide $\frac{1}{\sqrt{5}+\sqrt{2}}$ by $\sqrt{5}-\sqrt{2}$

$$\frac{1}{\sqrt{5}+\sqrt{2}} \text{ x} \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}}$$

$$= \frac{\sqrt{5}-\sqrt{2}}{(\sqrt{5})^2-(\sqrt{2})^2}$$

$$= \frac{\sqrt{5}-\sqrt{2}}{5-2}$$

$$= \frac{\sqrt{5}-\sqrt{2}}{3}$$

(iii) $\frac{1}{\sqrt{7}-2}$

Conjugate of $\sqrt{7}-2$ is $\sqrt{7}+2$

Multiply and Divide $\frac{1}{\sqrt{7}-2}$ by $\sqrt{7}-2$

$$\frac{1}{\sqrt{7}-2} \text{ x} \frac{\sqrt{7}+2}{\sqrt{7}+2}$$

$$= \frac{\sqrt{7}+2}{(\sqrt{7})^2-(2)^2}$$

$$= \frac{\sqrt{7}+2}{7-4}$$

$$= \frac{\sqrt{7}+2}{3}$$

Excercise 1.6

1.Find

(i) $64^{\frac{1}{2}} = (8^2)^{\frac{1}{2}} \quad = 8$

(ii) $32^{\frac{1}{5}} \quad = (2^5)^{\frac{1}{5}} = 2$

(iii) $125^{\frac{1}{3}} \quad = (5^3)^{\frac{1}{3}} = 5$

2.Find

(i) $32^{\frac{2}{3}} \quad = (2^5)^{\frac{2}{5}} \quad = 2^2 \quad = 4$

(ii) $9^{\frac{3}{2}} \quad = (3^2)^{\frac{3}{2}} \quad = 3^3 \quad = 29$

(iii) $16^{\frac{3}{4}} \quad = (2^4)^{\frac{3}{4}} \quad = 2^3 \quad = 8$

(iv) $125^{\frac{-1}{3}} = (5^3)^{\frac{-1}{3}} = 5^{-1} = \frac{1}{5}$

3. Simplify

(i) $2^{\frac{2}{3}} \times 2^{\frac{1}{5}}$ $\quad a^m \times a^n = a^{m+n}$

$= 2^{\frac{2}{3}+\frac{1}{5}} = 2^{\frac{10+3}{15}} = 2^{\frac{13}{15}}$

(ii) $\left(\frac{1}{3^3}\right)^7$

$= (3^{-3})^7 = 3^{-3 \times 7} = 3^{-21} = \frac{1}{3^{21}}$

(iii) $\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}}$ $\quad \frac{a^m}{a^n} = a^{m-n}$

$= 11^{\frac{1}{2}-\frac{1}{4}} = 11^{\frac{2-1}{4}} = 11^{\frac{1}{4}}$

(iv) $7^{\frac{1}{2}} \times 8^{\frac{1}{2}}$ $\quad (ab)^m = a^m \times b^m$

$= (7 \times 8)^{\frac{1}{2}} = 56^{\frac{1}{2}}$